Tracker-Based Adaptive Schemes for Optimal Waveform Selection

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***Abstract*—Gains in processing power and advances in radar signal processing enable waveform diversity. It has been demonstrated that much can be gained with waveform diversity when previous knowledge about the environment is available, or through adaptive clutter estimation. For tracking systems, it has been also demonstrated that waveform variations in response to tracker outputs can also be beneficial. The paper focuses on the latter line of research, presenting a survey and new results pertaining to adaptation for waveform selection in tracking systems. In particular, we introduce a golden standard for adaptive methods, and show that the golden standard is satisfied for a simple adaptive law. Numerical simulations are presented to illustrate the main results in the paper.**

I. Introduction

Gains in processing power and advances in radar signal processing enable waveform diversity, understood as the ability of changing the transmitted waveform across multiple time scales. It has been demonstrated that much can be gained with waveform diversity by making use of previous knowledge about the environment or through adaptive clutter estimation [1], [2]. It has been also demonstrated for tracking systems that waveform variations in response to tracker outputs can be beneficial as well [3]. Interestingly, analytical justifications for the latter have been given in terms of steady state behavior [4]. This brings to mind the state of adaptive control theory in the 1980s where interest was focused on adaptively controlling an unknown, but constant linear time invariant (LTI) dynamical system in the absence of disturbances, either deterministic or stochastic [5]. The golden standard at the time was to prove boundedness of the states of the extended system (LTI system plus controller), and convergence to zero of the error between the system's output and the output of a known reference model. Tracing a parallel with 1980's adaptive control theory, we introduce a golden standard for feedback-based waveform selection in tracking systems. Simply put, the golden standard is that an adaptive scheme should perform in steady state as well as the *best* system using a constant waveform. The implication is that the adaptive system should converge to the best choice of a fixed waveform. Notice that we are asking here much more than it was asked in adaptive control theory. Surprisingly, among other results, we determine conditions under which a simple adaptive scheme introduced in [6] satisfies the golden standard (Remark 2). In light of recent results from the control theory literature showing gains in performance with periodic sensor switching [7], we also investigate how waveform switching schemes, proposed in [8] (see also [9] and [10]), fare against adaptive rules and fixed waveforms. The paper is organized as follows. Section II introduces the notation and the systems under study. Section III presents a survey of representative tracker-based adaptive schemes proposed in the literature. Starting with a precise definition for the golden standard, Section IV contains our main theoretical results. Section V presents simulations illustrating the various results. Section VI closes the paper with our conclusions.

II. Notation and Preliminaries

We assume a discrete white noise acceleration model ([11], p. 273) $x_{k+1} = Fx_k + w_k$ where the process noise w_k is white with covariance $\mathbb{E}[w_k w_k^T] = Q$ and zero mean. The outputs are range and range rate, given by $y_k = Hx_k + v_k$ where the measurement noise v_k is white with covariance $\mathbb{E}[v_k v_k^T] = R_k$ and zero mean. Here,

$$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix},\ H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ Q = \sigma_w^2 \begin{bmatrix} \frac{T^4}{4} & \frac{T^3}{2} \\ \frac{T^3}{2} & T^2 \end{bmatrix},$$

and T denotes the sampling interval. Range and range-rate measurements are obtained using two types of pulses: constant frequency (CF) Gaussian pulses and linear frequency modulated (LFM) Gaussian pulses. Following [6] we assume the measurement noise covariance is given by:

$$R = \begin{bmatrix} r_{11} & r_{12} \\ r_{12} & r_{22} \end{bmatrix} = \begin{bmatrix} \frac{c^2\lambda^2}{2\eta} & -\frac{c^2 b\lambda^2}{\omega_0\eta} \\ -\frac{c^2 b\lambda^2}{\omega_0\eta} & \frac{c^2}{\omega_0{}^2\eta}\left(\frac{1}{2\lambda^2} + 2b^2\lambda^2\right) \end{bmatrix} \quad (1)$$

Here c denotes the wave speed (m/s), η denotes the SNR, $f_0 = \frac{\omega_0}{2\pi}$ denotes the carrier frequency (Hz), $\lambda > 0$ denotes the pulse length (s) and b denotes the sweep rate (Hz/s). b can be positive (LFM upsweep), negative (LFM downsweep) or zero (CF). Notice that $\det(R) = \frac{c^4}{4\omega_0^2\eta^2} = d$ does not depend on the parameters λ or b. We assume the outputs are processed by a Kalman Filter with the covariance update equations given as follows [12]:

$$\begin{aligned} S_k &= P_{k/k-1} + R_k \\ W_k &= P_{k/k-1} S_k^{-1} \\ P_{k/k} &= P_{k/k-1} - W_k S_k W_k^T \\ P_{k+1/k} &= F P_{k/k} F^T + Q \end{aligned}$$

Let $\{R_k\},\ k = 1, 2, \cdots$, denote a sequence of measurement noise covariance matrices, obtained by changing λ and b. When it exists, let $P_\infty(\{R_k\})$ denote the limit as the time index k goes to infinity of the time-varying Riccati equation associated with the sequence $\{R_k\}$. When $R > 0$ is constant, we have an algebraic Riccati equation with positive definite

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solution denoted as $P_\infty(R)$, since F and Q satisfy the conditions in [11], p. 211. When the sequence $\{R_k\}$ is periodic with period N, the time-varying Riccati equation converges to a periodic solution with period N [13] under conditions which are satisfied by F and Q.

III. Tracker-based adaptive schemes

Table I summarizes a number of representative articles published in the past 20 years in the subject of adaptation for waveform diversity using tracker outputs.

Articles [14] and [15] are close in spirit, addressing air-to-air scenarios with multiple well-separated targets. The overall objective is to control the total energy transmitted to a target so that the largest number of targets can be tracked within a certain level of uncertainty. [15] utilizes several interrelated performance measures: the mean revisit intervals, the mean number of dwells for a successful update, the mean number of sensor allocations required for track maintenance (as in [14] the sensor is given multiple chances to reacquire a target) the mean energy spent for successful reacquisition, the mean energy spent for track maintenance, and the mean RCS of the targets estimated during tracking.

In [6] closed form expressions are obtained for the controlled variables (pulse length and sweep rate) as a function of the tracker covariances. Two performance indices are considered: the volume of the validation gate (as noted in Table I) and the tracking error, measured as the trace of the state error covariance. A greedy approach is utilized as described in section IV. A later paper [16] addressed the case where false measurements due to clutter are present. Performance improvements are shown in [16] using waveform selection plus PDAF in contrast with PDAF alone.

In [4] the authors address the combined problem of waveform selection and optimization of the detection threshold. Semi-analytical schemes are employed to analyze the performance of the PDAF filter, and comparison with the results obtained with the best fixed waveforms is presented. This comparison is central to the golden standard described on this section.

In [17] a multi-target scenario is considered, and the choice of the target to be illuminated on a given epoch is controlled through the pointing direction. One-step ahead and two-steps ahead grid searches are performed in order to obtain the best waveform parameters (following the model in [6]) and track to be illuminated. Experiments show that two-steps ahead leads to better performance.

In contrast with [6] and [4], [18] utilizes a nonlinear measurement model and particle filters to deal with the nonlinearity. The optimal waveforms are selected via a grid search on the controlled variables across a suitable time interval.

[19] is a departure from the earlier works in three ways: (1) multiple sensors are employed and the measured quantities are the bounced signals received at each sensor from transmissions from other sensors; (2) an extended state space combining the motions of the target and the time varying channel is employed; (3) The controlled variables are the sensors utilized for each target and the corresponding power. To the best of our knowledge, [19] is the first article which uses adaptation to jointly estimate channel state (as in [2]) and target state.

In [20] the authors encode prior information about the tracking tasks on specially designed memory blocks utilizing multi-layer neural networks. According to the authors, the presence of these multiple memories endows the overall system with cognition capabilities. Several numerical experiments demonstrate the superiority of the resulting system over other schemes, including an adaptive scheme.

IV. Main results

While we do not include false measurements in our theoretical studies and numerical studies, our metric of performance is directly tied to counteracting the effects of false measurements in tracking performance. Namely, we attempt to minimize the volume of the validation gate, which in the case of Kalman filters is directly proportional to the square root of the determinant of the measurement noise covariance [23], p. 169. This allows a more precise definition for the golden standard than the one offered in the Introduction:

Definition 1 (The golden standard): We say that a time-varying sequence of waveforms each with constant determinant d satisfies the golden standard if $P_\infty(\{R_k\})$ exists, and $\det(P_\infty(\{R_k\}) + R_k) = \det(P_\infty(R^*) + R^*)$, where $R^* = \arg\min_{\{R:det(R)=d\}} \det(P_\infty(R) + R)$ ■

Clearly, if the metric of performance was different (say, the tracking error) a similar golden standard could be defined in terms of the new metric. Out of the eight articles surveyed in Section III only [4] compares the performance of the adaptive system with the performance of the best fixed waveform.

Theorem 1 (Greedy optimal (GO) CF pulses): Let us consider CF pulses, i.e. $b = 0$ (or $r_{12} = 0$) in equation (1). While we seek to minimize $\det(P_\infty(R^*) + R^*)$, adaptation decisions must be made at each time step. Following [6] we utilize a greedy one step optimization scheme where $R_k^{CF} = \begin{bmatrix} r_{11}^* & 0 \\ 0 & r_{22}^* \end{bmatrix} = \arg\min_{\{(r_{11},r_{22}):r_{11}r_{22}=d\}} \det(P_{k/k-1}+R_k)$. Such greedy schemes have the advantage of allowing for the derivation of closed form solutions, and relative simplicity of implementation. Greedy schemes may be optimal in very simple cases [24], but they are not optimal in general [25]. Defining $P_{k/k-1} = \begin{bmatrix} p_{11_k} & p_{12_k} \\ p_{12_k} & p_{22_k} \end{bmatrix}$ it is straightforward to show that $R_k^{CF} = \begin{bmatrix} \sqrt{\frac{p_{11_k} d}{p_{22_k}}} & 0 \\ 0 & \sqrt{\frac{p_{22_k} d}{p_{11_k}}} \end{bmatrix}$ is a point of global minimum. Indeed, dropping the time indexes, let $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$ represent an arbitrary positive definite matrix, and $R = \begin{bmatrix} r_{11} & 0 \\ 0 & \frac{d}{r_{11}} \end{bmatrix}$ with $r_{11} > 0$ represent an arbitrary 2×2 diagonal matrix with determinant equal to $d > 0$. Let $f(r_{11}) = \det(P + R)$ and we seek r_{11}^* which minimizes $f(\cdot)$. Now,

$$\frac{df(r_{11})}{dr_{11}} = p_{22} - \frac{dp_{11}}{r_{11}^2}$$

Article	Target Model	Tracker	Measured Variables	Controlled Variables	Metric	Remarks
van Keuk and Blackman [14]	Singer 3D [21] p.200	PDAF [21] p. 350	Angles and range	Time-on-target and pointing direction	Load in the system	Includes target search
Kershaw and Evans [6]	WPAM 1D [11] p. 271	Linear Kalman filter	Range and range rate	Pulse length and sweep rate	Validation gate volume	Closed-form expressions
Koch [15]	IMM 3D [21] p. 221	MHT [21] p. 1069	Angles, range and amplitude	Energy/dwell and pointing direction	Several (see text)	Includes RCS estimation
Suvorova, Mušicki, Moran, Howard and La Scala [17]	IMM 2D (multiple targets)	LMIPDA [22]	Range and range rate	Pulse length, sweep rate and pointing direction	Tracking error (MSE)	One-step and two-steps ahead grid search
Hong, Evans and Shin [4]	WNAM 1D [11] p. 270	PDAF	Range and range rate	Pulse length, sweep rate and amplitude	Tracking error (MSE)	Includes detection threshold optimization
Sira, Papandreou-Suppappola and Morrell [18]	WNAM 2D	Particle filter	Range and range rate	Pulse length and sweep rate	Tracking error (MSE)	Optimization uses grid search
Chavali and Nehorai [19]	WNAM 2D (multiple targets)	Extended state space (targets and channels)	Reflected signals (see text)	Antenna subset and power	Tracking error (MSE)	Multi-sensor, multi-target
Haykin, Xue and Setoodeh [20]	Coordinated turn 2D [21] p. 205	Cubature Kalman filter	Angle and range	Pulse length and sweep rate	Tracking error (MSE)	Uses prior information encoded in memory

TABLE I. REPRESENTATIVE ARTICLES ADDRESSING TRACKER-BASED WAVEFORM ADAPTATION.

and
$$\frac{d^2 f(r_{11})}{dr_{11}^2} = \frac{2dp_{11}}{r_{11}^3} > 0$$
for all $r_{11} > 0$, verifying our claim. ■

Theorem 2 (Greedy optimal LFM pulses): For the case where $b \neq 0$ (or $r_{12} \neq 0$) it is possible to show that
$$\begin{aligned} R_k^{LFM} &= \sqrt{\frac{d}{p_{11_k} p_{22_k} - p_{12_k}^2}} \times \begin{bmatrix} p_{11_k} & p_{12_k} \\ p_{12_k} & p_{22_k} \end{bmatrix} \\ &= \sqrt{\frac{d}{\det(P_{k/k-1})}} P_{k/k-1} \end{aligned}$$
is a point of global minimum. Indeed, let us redefine $R = \begin{bmatrix} r_{11} & r_{12} \\ r_{12} & \frac{d+r_{12}^2}{r_{11}} \end{bmatrix}$ with $r_{11} > 0$, representing an arbitrary 2×2 positive definite matrix with determinant equal to $d > 0$. Let $g(r_{11}, r_{12}) = \det(P + R)$, and we seek a pair (r_{11}, r_{12}) which minimizes $g(r_{11}, r_{12})$. Now,
$$\frac{\partial g(r_{11}, r_{12})}{\partial r_{11}} = p_{22} - \frac{p_{11}(r_{12}^2 + d)}{r_{11}^2},$$
$$\frac{\partial g(r_{11}, r_{12})}{\partial r_{12}} = \frac{2p_{11} r_{12}}{r_{11}} - 2p_{12},$$
with corresponding Hessian matrix given by:
$$H(r_{11}, r_{12}) = \begin{bmatrix} \frac{2p_{11}(r_{12}^2+d)}{r_{11}^3} & -\frac{2p_{11} r_{12}}{r_{11}^2} \\ -\frac{2p_{11} r_{12}}{r_{11}^2} & \frac{2p_{11}}{r_{11}} \end{bmatrix}$$
Now,
$$\det(H(r_{11}, r_{12})) = \frac{4dp_{11}^2}{r_{11}^4} > 0$$
for all (r_{11}, r_{12}) with $r_{11} > 0$ verifying our claim. ■

Remark 1 (Theorem 1, Theorem 2 and [6]): Theorem 1 and Theorem 2 could have been derived using the developments in [6] leading to the computation of optimal values for b and λ. We followed a different route, minimizing the matrices directly in terms of their entries, not in terms of specific parameters ■

Theorem 3 (A quasi-optimal fixed CF pulse): From [26] Theorem 3.3 we have: $P_\infty(R) < FRF^T + Q$ for non-singular F. Hence, instead of minimizing $\det(P_\infty(R) + R)$ it makes sense to determine R with fixed determinant d which minimizes the upper bound $\det(FRF^T + Q + R)$. Following the same type of arguments used in the proofs of Theorem 1 and Theorem 2 it is possible to show that the (quasi)-optimal solution is a diagonal matrix obtained by solving the depressed cubic
$$2\sigma_w^2 r_{11}^3 - \frac{\sigma_w^2 T^2 d}{2} r_{11} - 2d^2 = 0$$
for $r_{11} > 0$ and writing $r_{22} = \frac{d}{r_{11}}$ ■

Remark 2 (Adaptation using GO pulses): Extensive numerical simulations have shown that the sequence $\{R_k^{CF}\}$ satisfies the golden standard. Interestingly, simulations have also revealed that the sequence $\{R_k^{LFM}\}$ does not. While we do not have formal proofs for the convergence of the corresponding time varying Riccati equations, we present plausibility arguments for the superiority of the CF waveform based on the bounds in $P_\infty(R)$ utilized in Theorem 3. We note at first that given a fixed positive definite matrix $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$ and their corresponding LFM and CF sensor matrices

$$R_1 = \sqrt{\frac{d}{p_{11}p_{22} - p_{12}^2}} \times \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}, \quad \text{and}$$

$R_2 = \begin{bmatrix} \sqrt{\frac{p_{11}d}{p_{22}}} & 0 \\ 0 & \sqrt{\frac{p_{22}d}{p_{11}}} \end{bmatrix}$ we have

$$\det(P + R_2) - \det(P + R_1) =$$
$$2d^{1/2}\left[(p_{11}p_{22})^{1/2} - (p_{11}p_{22} - p_{12}^2)^{1/2}\right] > 0$$

showing that at any given time, the LFM sensor will reduce the performance index the most. However, we are interested in the *successive* application of the sensors, in which case we are left with the problem of establishing convergence of the resulting iterations. We now assume the iterations converged, and take the bounds in Theorem 3 as approximations, i.e.:

$$P_\infty(R_1) \approx X_1 = FR_1F^T + Q$$
$$P_\infty(R_2) \approx X_2 = FR_2F^T + Q$$

Now:

$$\det(X_1 + R_1) - \det(X_2 + R_2) =$$

$$\frac{T^4}{2}d^{1/2}\sigma_w^2 p_{22}^{1/2}\underbrace{\left[\frac{p_{22}^{1/2}}{(p_{11}p_{22} - p_{12}^2)^{1/2}} - \frac{1}{p_{11}^{1/2}}\right]}_{(>0)} +$$

$$2T^2d^{1/2}\sigma_w^2 p_{11}^{1/2}\underbrace{\left[\frac{p_{11}^{1/2}}{(p_{11}p_{22} - p_{12}^2)^{1/2}} - \frac{1}{p_{22}^{1/2}}\right]}_{(>0)} +$$

$$T^2dp_{22}\underbrace{\left[\frac{p_{22}}{p_{11}p_{22} - p_{12}^2} - \frac{1}{p_{11}}\right]}_{(>0)} > 0$$

suggesting that successive applications of the LFM sensor will lead to a larger metric of performance than successive applications of the CF sensor, i.e. CF should be preferred. ■

V. Numerical results

We consider the sonar set-up from [9], p. 473: $T = 30$ s, $c = 1500$ m/s, $\frac{\omega_0}{2\pi} = f_0 = 3.5$ kHz, SNR $= 20$ dB ($\eta = 100$). As σ_w^2 varies in the range of 1.0×10^{-6} to 1.0×10^{-3} we computed four quantities:

1) The value of λ which minimizes $\det(P_\infty(R) + R)$ with $b = 0$. This is performed through a grid search. We found that $b = 0$ minimizes $\det(P_\infty(R) + R)$ for all optimal λ, i.e. we actually performed a two dimensional grid search along λ and b.
2) The quasi-optimal value of λ obtained by following the procedure in Theorem 3.
3) The constant value of λ obtained by iterating the adaptive procedure in Theorem 1 till convergence.
4) The value of λ which minimizes $\det(P_\infty(R_k^P) + R_k^P)$ where $\{R_k^P\}$ is a 2-periodic sequence obtained by switching R in equation (1) between (λ, b) and $(\lambda, -b)$. Here, λ and b are determined through a two dimensional grid search. $\det(P_\infty(R_k^P) + R_k^P)$ converges to a 2-periodic sequence. For this reason, we compute the mean value of the sequence.

Figure 1 shows the values of λ in the first three cases and the corresponding limiting values of $\det(P_\infty(R_k) + R_k)$. A few remarks are in order:

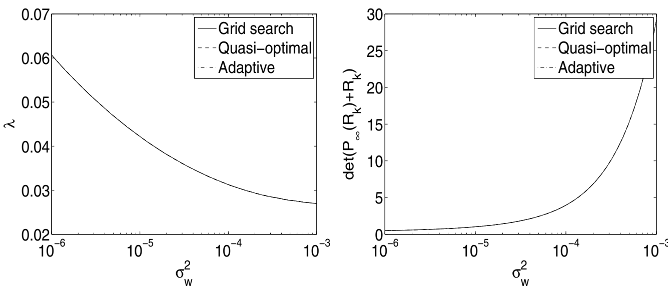

Fig. 1. Values of λ and limiting values of $\det(P_\infty(R_k) + R_k)$.

- As noted in Remark 2 the steady state for adaptation *coincides* within numeric tolerance with the best choice for λ obtained through grid search.
- The quasi-optimal solution performs extremely well for the whole range of σ_w^2 utilized. The actual value of λ selected by the quasi-optimal strategy is extremely close to the one obtained through grid search and adaptation.
- For the fourth case considered (switching between (λ, b) and $(\lambda, -b)$), grid search revealed that $b = 0$ is the optimal choice. This is in contrast with [8] where switching between an upsweep LFM waveform and a downsweep LFM waveform was recommended, on basis of a different metric of performance.

VI. Conclusions

Waveform diversity enables three forms of adaptation: (1) using prior information about the environment to change waveforms; (2) use the transmitted waveforms as a scheme for sensing the environment and performing clutter estimation; (3) use information from the tracker to perform adaptation. This paper addressed the third form of adaptation. A survey and new results were presented, including the formulation of a golden standard for adaptive schemes. It was shown through analytical developments and simulations that a simple adaptive law proposed in [6] satisfies this golden standard. We see two directions of research motivated by our survey and results. The first direction is to deal with multi-target scenarios with target confusion. So far, multi-target studies [14], [15], [17], [19] deal with the case where targets are far apart. This simplifies the problem, since we can associate a particular dwell with a particular target. In the presence of confusion this is not possible, and multi-target techniques such as MHT

(Multiple Hypothesis Tracking) [21], p. 1069 and JPDA (Joint Probabilistic Data Association) [21], p. 359 need to be used. In summary, the formulation of an adaptive waveform selection scheme in the presence of confusion represents a fruitful area of research. The second direction is to combine tracker-based waveform selection with the two other forms of adaptation. [19] is a step towards this direction, but much more can be done when previous information is available.

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